Multi-objective Optimization for Synthesis of Mass Exchange Networks using a fuzzy approach

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Abstract

This paper introduces a novel multi-objective optimization method for synthesizing Mass Exchange Networks (MENs), which aims to integrate process streams containing pollutants with simpler or target streams to meet environmental standards. The core research problem addressed is the challenge of simultaneously minimizing operational costs and ensuring efficient pollutant removal, which is not fully addressed by traditional MEN design methods.

The proposed approach combines recent advances in pinch technology with fuzzy logic to determine the optimal network configuration. A multi-objective function is developed that considers critical factors such as the surplus capacity of the mass separation agent, the load removal by external separation agents, and the number of plates required in the process. The optimization method is designed to provide more balanced solutions, taking into account both environmental compliance and cost efficiency.

To validate the effectiveness of this method, two well-known case studies were conducted: coke-oven gas (COG) sweetening and phenol removal. The results demonstrate that the fuzzy logic-based optimization approach outperforms existing methods, offering more precise, cost-effective solutions while ensuring adherence to environmental standards.

In conclusion, this study presents a modern and efficient strategy for synthesizing MENs. The fuzzy logic-based optimization method offers engineers a practical tool to design systems that are both environmentally compliant and economically viable, marking a significant advancement in the field of process optimization for sustainable industrial applications.

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I. Introduction

Mass Exchange Networks (MENs) are critical for minimizing waste and improving cost-effectiveness in industrial processes, especially in feed preparation, product separation, and by-product recovery. MENs are vital for process synthesis, balancing environmental sustainability with optimizing operational costs (Total Annual Cost, TAC).

Originally, MENs were optimized using pinch technology, focusing mainly on energy efficiency and operating costs. However, this method overlooks other essential factors, such as separation efficiency and capital investment[1]. To address these gaps, several optimization techniques have been developed over time. These include two-stage Automated Synthesis (LP and MILP), proposed by [2], which minimizes operating costs but suffers from computational complexity, Mixed-Integer Non-Linear Programming (MINLP), introduced by [3], optimizing network structure integration but requiring considerable effort for fine-tuning, without guaranteeing optimal solutions. Also, State-Space and Superstructure Approaches, discussed by [4][5], improve integration and consider both capital and operational costs, but still lack effective environmental compliance integration.

Despite their effectiveness, these methods face challenges such as computational complexity and limited capacity to handle real-world dynamics, including fluctuating operational conditions.

This paper introduces a fuzzy logic-based multi-objective optimization approach to synthesize MENs, aiming to balance cost-efficiency and environmental performance while reducing the computational burden of traditional methods. The approach is validated through two widely recognized case studies, demonstrating its advantages over methods like pinch analysis and MINLP. The fuzzy logic approach simplifies the optimization process, making it more accessible to engineers and reducing the computational load compared to more complex methods.

The motivation behind this research is to overcome the limitations of traditional MEN optimization methods, which often focus only on operational efficiency or cost reduction. By integrating multiple objectives—pollutant removal efficiency, capital costs, and operational costs—into a single framework, the paper provides a comprehensive solution for MEN design.

The fuzzy optimization approach offers a flexible, computationally efficient solution for simultaneous optimization of multiple objectives. It ensures better environmental and cost performance, providing engineers with a more adaptable tool compared to traditional methods.

This approach also addresses real-world challenges, such as dynamic operational conditions, which previous methods have not sufficiently accounted for.

The significance of this study lies in its provision of a user-friendly tool for designing cost-effective and environmentally compliant MENs. By integrating multiple optimization objectives, the approach offers a more holistic solution than traditional methods, contributing to improved sustainability and reduced operational costs.

The results from the case studies show clear improvements in both cost and environmental performance, compared to traditional methods. This research fills a significant gap in the literature by introducing a multi-objective approach to MEN synthesis, an area that previous studies have not fully addressed.

II. Methodology

The following is a MEN synthesis concerns discussed in this paper: Given a set of rich streams R_i, their flow rates G_i (i = 1,2,..., N_R) in which some transferable components are to be removed, the input and output compositions of components C (C=1,2,…,N_C), $Y_{i,c}^{in}$, $Y_{i,c}^{out}$ as well as a set of lean streams S_j, their flow rates L_j (j = 1,2,….,N_s) to be it is desired to create a network of mass exchange units capable of transferring a set of specific species from rich to lean streams while minimizing TAC.

A. Corresponding composition scales

To determine the composition relationships between the rich and lean streams, the concept of "corresponding composition scales" comes in handy. There is an equilibrium relationship between the ith rich stream and the jth lean stream that is frequently presumed to be linear and stated as

$$Y_i = m_j x_j + b_j \qquad (1)$$

Where m_j and b_j are the equilibrium constants. If the ith rich stream composition is yi, the theoretically feasible utmost composition of the MSA is x, according to Eq.(l). To avert the unlimited size of mass exchangers, the minimal composition difference E must be used. As a result, the linear equation that takes ε into account is written as

$$Y_i = m_j(x_j + \varepsilon_j) + b_j \qquad (2)$$

As a result, for a given Y_i, the value of refers to the maximum composition that is practically achievable in the jth lean stream; similarly, for x_j, the value of Y_i corresponds to the minimum

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composition of the pollutant in the ith rich stream that is required to practically transfer the component from rich streams to MSAs. To produce a composition interval diagram (CID) consisting of a sequence of "composition intervals" that correspond to the source or target component composition for each stream.

The whole composition range is expected to be split into n composition breaks in the CID, with the highest composition interval represented as k=1 and the lowest as k=n.

The mass exchange of the ith rich stream and the jth lean stream travelling through the kth break can be estimated as follows:

$$G_I \times \Delta y_{i,k} = L_j \times \Delta x_{j,k} \tag{3}$$

B. Economic analysis

The economic evaluation aims to estimate the yearly operational and capital costs for the network design. Annual operating costs are determined based on the quantity of MSAs used in the system. The formula for calculating the Annual Operating Cost (AOC) in dollars per year is expressed as:

$$AOC = C_{msa}*L*t \tag{4}$$

Where:

- **L**: MSA flow rate (kg/s),
- **C_{msa}**: Cost per kilogram of MSA ($/kg), and
- **t:** Operating time in second.

Additionally, capital costs for all mass transfer equipment, such as columns, are calculated using a simplified correlation with the number of stages. This approach provides a quick way to evaluate different stream configurations, like varying S3 placements, without the need for complex computations. **The Kremser equation** is employed to estimate the number of equilibrium steps (N_e) required, based on the absorption factor (A_f).

For $A_f \neq 1$

$$N_e = \frac{ln\left[\left(\left(Y_{ij}^{in}-m_jX_{ij}^{in}-b_j\right)/\left(Y_{ij}^{out}-m_jX_{ij}^{in}-b_j\right)\right)\left(1-(1/A)\right)+(1/A)\right]}{ln(A)} \tag{5}$$

For $A_f = 1$

$$N_e = \frac{\left(Y_{ij}^{in}-Y_{ij}^{out}\right)}{\left(Y_{ij}^{out}-m_jX_{ij}^{in}-b_j\right)} \tag{6}$$

Where

$$A_f = \frac{L}{m_{ij} G_i} \tag{7}$$

Y_{ij}^{in}, Y_{ij}^{out}, and X_{ij}^{in} refer to the input and outlet compositions of the appropriate components of the ith rich stream and the jth lean stream going through the mass exchanger, then the system's total annualized cost (TAC) is given by :

Total Annual Cost =Annual Operating Cost + Annual fixed Cost   (8)

C. Fuzzy set theory:

Fuzzy set theory was first formulated by [2], and theoretical information is available in [3]. It will only discuss the fundamental concepts of such a theory and its usual applications in chemical engineering. Fuzzy set theory can describe uncertainty in chemical engineering in a variety of ways.

[4] differentiated between two types of uncertainty: ambiguity and imprecision. A proposition is ambiguous if it is true or false and cannot be determined conclusively. A proposition is imprecise if its value cannot be defined appropriately concerning a certain scale. Both ambiguity and imprecision can be classified into a variety of uncertainty types. As a result of the lack of precision in terms like "high" or "small," the uncertainties of heuristic rules here are of the latter kind.

A fuzzy set A in the space $X = \{x\}$ can be defined as the set:

$$A = \{x, \mu_A \ (x)\} \qquad\qquad \forall \ x \in X \qquad\qquad \mu_A : \rightarrow [0, 1]$$

Where : $\mu_A \ (x)$ Expresses the grade of membership of x in A

$\mu_A \ (x) = 0$ Means that x is an indefinitely a member of A

$\mu_A \ (x) = 1$ Means that x is definitely a member of A

The intermediate values of the membership function denote partially defined, to some extent, membership of A. In brief, the theory of fuzzy sets is a step towards unification between the accuracy of classical mathematics and the general imprecision of reality, an agreement born of humanity's never-ending quest for a deeper understanding of brain processes and cognition. [5].

Some algebraic operations on fuzzy sets can be defined by [3], like

Union $\mu_A \ \cup B \ = max \ (\mu_{A,} \mu_B)$

(9)

Intersection $\mu_A \ \cap B = \min(\mu_{A,} \mu_B)$ (10)

The decision is to be made by evaluating all the related rules at different levels in a knowledge base. The evaluations are carried out according to the MAX-MIN algorithm.

$$\boldsymbol{\mu_j}(x) = \boldsymbol{max_{i \in I}} \{\boldsymbol{min_{k \in K}} \{\boldsymbol{\mu_{i1}}(x_1)\}, \boldsymbol{\mu_{i1}}(x_1), \boldsymbol{\mu_{ik}}(x_k), \boldsymbol{\mu_{i1k}}(x_k)\} \tag{11}$$

Where $\boldsymbol{\mu_{ik}}(x)$: Membership function of variable x in fuzzy set k representing the kth antecedent of the ith rule at the jth level.

The MIN operation yields a set of truth values through the evaluation of the membership functions of all the rules. Then, a single rule is selected by performing the MAX operation, i.e.

$$\tau_i = \min \{\mu_{i1}(x_1)\}, \mu_{i2}(x_2), \mu_{ik}(x_k)\} \tag{12}$$

Then,

$$\tau = \max\{\tau_1, \tau_2, \ldots \ldots \ldots, \tau_I\} \tag{13}$$

This selected rule is activated or fired. The same operation is repeated at the succeeding level based on the information received from the preceding level.

D. Fuzzy approach strategy

The same procedure is repeated at the succeeding level depending on the information acquired from the preceding level, and the following algorithm was developed to select optimal mass exchanger networks based on the strategy of fuzzy set theory [6,7], which it consists of three steps:

Step1. Normalization decision matrix for each value according to the following rules:

Rule (1): if the excess capacity of process MSA (S_1) and the number of plates in the column is to be increased, the fuzzy quantities are represented by the following membership function:

$$\mu_i = \begin{cases} 0, & if.f \leq f_{min} \\ \frac{f - f_{min}}{f_{max} - f_{min}}, & if.f_{max} > f > f_{min} \\ 1 & if.f \geq f_{max} \end{cases} \tag{14}$$

Rule (2): if the load to be removes by external MSA (S_2) is to be decreased; the fuzzy quantities represented by the following membership function:

$$\mu_j = \begin{cases} 0, & if.f \geq f_{max} \\ \frac{f-f_{max}}{f_{min}-f_{max}}, & if.f_{max} > f > f_{min} \\ 1 & if.f \leq f_{min} \end{cases} \tag{15}$$

Step2. Calculate the minimum value of parameters according to equation (12).

Step3. The final step corresponds to the choice of the greater numerical value (the mass transfer load at the exchange unit); the operation is carried out by comparing (τ) values for all optimum minimum composition approach $(\mathcal{E})$ and by choosing the greatest one according to equation (13). The procedure will show in Fig (1)
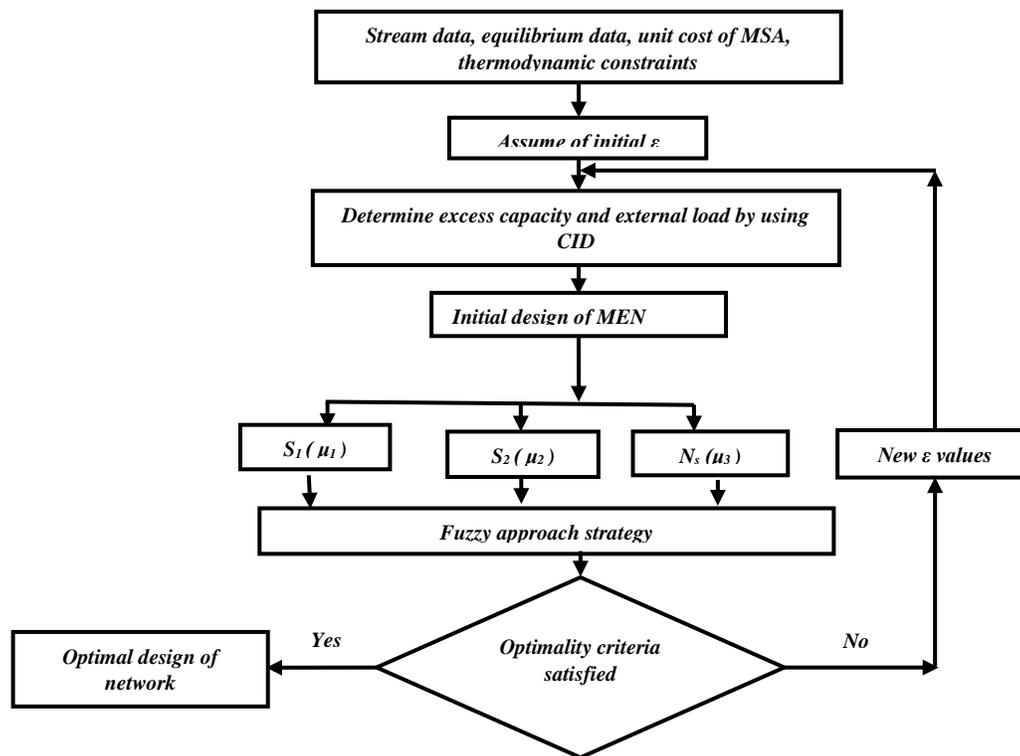

Figure (1) Optimized procedure for MEN flowchart

III. CASE STUDIES:

The proposed method will be tested using two problems reported in the literature for comparison

A. *Case study (1)*: Coke – Oven Gas [COG] sweetening which reported by [1]

A sample problem for the sweetening of coke-oven gas (COG) is deduced by [1] . Additionally, it has recently been looked into by such as [8] . The method's main objective is to eliminate hydrogen sulfide from COG. The example takes into account two MSA streams (S_1, S_2) and two rich streams (R_1, R_2).

Tables 1 and 2 provide the information and display the component supply and goal concentrations. Rich stream mass flow rates and the upper flow rate bounds of MSAs are both provided.

Table 1. Case study (1) data stream

Rich stream			
Stream	G_i	Y_i^{in}	Y_i^{out}
R_1	0.9	0.07	0.0003
R_2	0.1	0.051	0.0001

Lean stream			
Stream	L_j	X_i^{in}	X_i^{out}
S_1	2.3	0.0006	0.0310
S_2	∞	0.0002	0.0035

Table 2. The Equilibrium relation and cost for case study (1)

Stream	Equilibrium relation	Cost ($\$year^{-1}$)/($kgsec^{-1}$)
hydrogen sulfide aqueous ammonia (S_1)	$Y_1 = 1.45 X_1$	117360
Methanol (S_2)	$Y_2 = 0.26 X_2$	176040

The two solvents and the annualized investment cost of such a column are examined for plate columns. $600 N^{.74}\$$ where N_s is the theoretical plates number in the column. Compute the

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composition relations between the rich and lean streams, the extra capability of the process MSA (S_1), the load that has to be removed by the external MSA (S_2), and the number of plates in the column as indicated in Table 3 by assuming the various the minimal composition values method ($\mathcal{E}$).

Table 3. Values of $(S_1, S_2, N_{s)}$ for different minimum composition approaches

Minimum composition approach ($\mathcal{E}$)	S_1	S_2	Number of plates ($N_{s)}$
0.00040	2.192	0.355	25
0.00035	2.195	0.333	25
0.00030	2.196	0.312	25
0.00025	2.199	0.288	25
0.00020	2.202	0.267	28
0.00015	2.204	0.245	30
0.00010	2.207	0.224	33

To select the optimum minimum composition approach ($\mathcal{E}$), we use the Fuzzy approach. For each function, we calculate the normalized decision matrix for different ($\mathcal{E}$) by using equations. Table 4 shows the results.

Table 4. Normalized decision matrix of MEN at different ($\mathcal{E}$) for case study 1

Minimum composition approach ($\mathcal{E}$)	S_1 (μ_1)	S_2 (μ_2)	Number of plates ($N_{s)}$ (μ_3)	Min	Max
	Normalized decision matrix				
0.00040	1.000000	0.000000	1.00000	0.000000	
0.00035	0.800000	0.167939	1.00000	0.167939	
0.00030	0.733333	0.328244	1.00000	0.328244	
0.00025	**0.533333**	**0.511450**	**1.00000**	**0.511450**	**0.511450**
0.00020	0.333333	0.671756	0.62500	0.333333	
0.00015	0.200000	0.839695	0.37500	0.200000	
0.00010	0000000	1.00000	0000000	0.000000	

The optimum solution is selected based on the maximum greater value (**0.511450**) which corresponds to ($\mathcal{E}$) = **0.00025**. Fig. 2 displays the optimal network structure where the mass transfer load at the exchange unit is indicated by the numerical value in parenthesis, while composition and/or flow rates are indicated by the other values. S_1 and S_2 each have flow rates of 2.199 and 0.288 kg/sec, respectively.

For optimal composition approach is selected to demonstrate the proposed method at ($\mathcal{E} = 0.00025$) the composition relations between the rich and lean streams as shown in table 5.

Table 5. The composition relations between the rich and lean streams.

Y	X
0.0700	0.04803
0.0510	0.03492
0.0453	0.0310
0.0012	0.0006
0.0003	0
0.0001	0

For calculation of the extra capability and load to be eliminated by using composition interval which shown in Table 6

Table 6. Composition interval diagram [CID] at ε = 0.00025 for case (1)

Interval		Y	X_1	$\Sigma G.\Delta Y - \Sigma L.\Delta X$	Cascade diagram	
	0	R₁ 0.07	0.04817	0	0	0.0028
Above					0.0171	0.0171
the pinch	1	0.051 R₂	0.0351	0.0171	0.0171	0.0198
					0.00569	0.00569
	2	0.04531	0.0310 S₁	0.006	0.0231	0.0258
					-0.0258	-0.0258
Pinch point	3	0.00102	0.00060	-0.0258	-0.0028	0
					0.0007	0.0007
Below the pinch	4	0.0003	0.000107	0.0007	-0.0021	0.0007
					0.00002	0.00002
	5	0.0001	0.0	0.00002	-0.0021	0.00072

From Table 6 excess capacity of process $= 0.00305 Kg/Sec$, then the actual amount of $S_1 = (2.3 - \frac{0.00258}{(0.031 - 0.0006)})$=2.208 Kg/Sec, similer the load to be removed= 0.00072, then the amount of S_2 required $= \frac{0.00072}{(0.0035 - 0.0002)}$ =0.218Kg/Sec, at the pinch point, the rich composition stream = 0.00102 and the lean composition stream =0.0006.

Table 7 compares different solutions, where [1] reported a network cost of (52.604 * 10^4) \$/Yr.) for minimum composition approach equal (0.0001) using the Pinch analysis method,[9] obtained a network cost of ((91.800 * 10^4) \$/Yr.) for minimum composition approach equal (0.0001) using

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MINLP, and [12] uses the Pinch analysis method, which gives a network cost of ((52.604 * 10^4) \$/Yr.) and It was discovered in this study that the cost optimal network has a minimum composition (0.00025) and network cost (\$ 42.257*10^4 \$/yr.
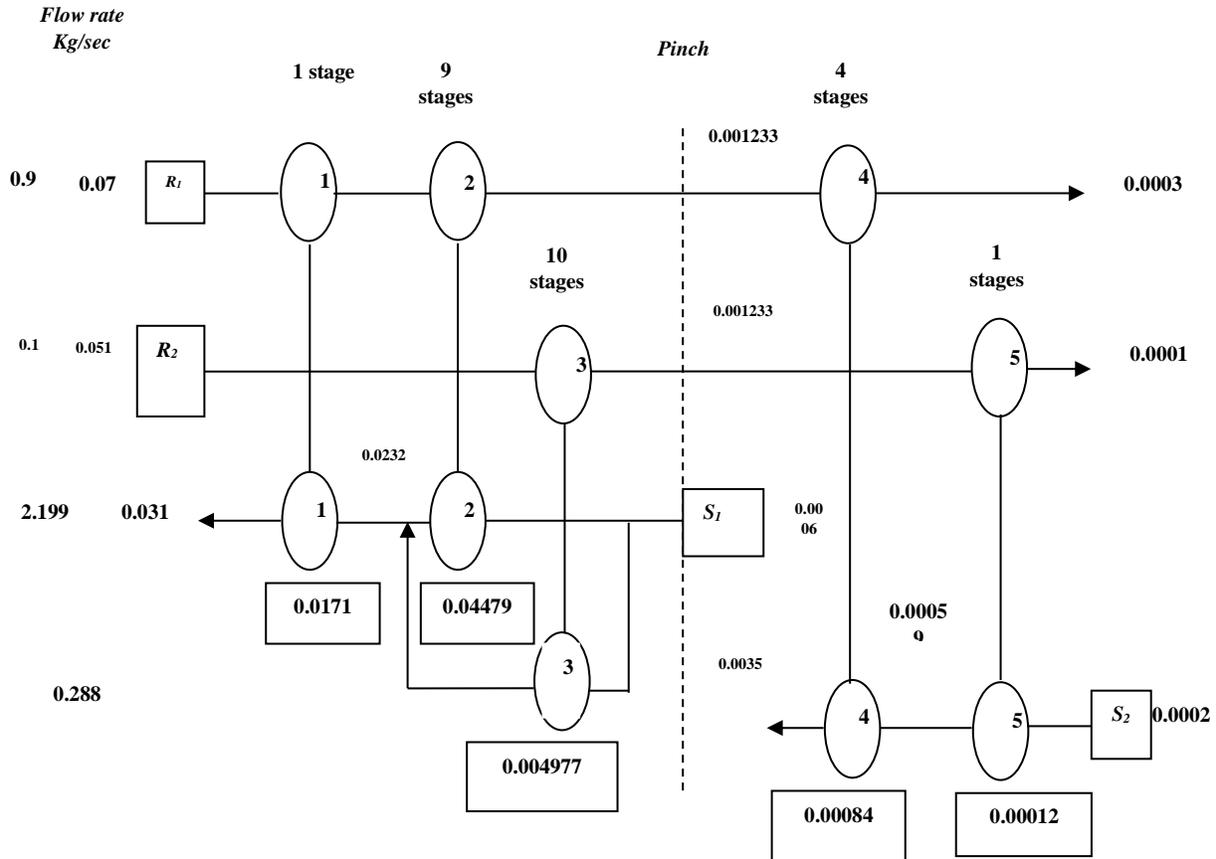

Figure (2) optimal network structure for case 1

Table 7. Comparisons of solution obtained by different authors for case study 1.

Author	used techniques	Minimum composition approach (ε)	Number of stages (N_{actual})	Annual fixed cost (\$/year)	Annual operating cost *10^4 (\$/year)	Total annual cost *10^4 (\$/year)
[1]	Pinch analysis	0.0001	50	227600	29.844	52.604
[9]	MINLP	0.0001	8	37213.2	88.079	91.800
[10]	Pinch analysis	0.0001	50	227600	29.844	52.604
[11]	Super-target method	0.00031	25	113800	31.326	42.706
Current work	Fuzzy approach	0.00025	25	113800	30.877	42.257

B. *Case study (2):* **Dephenalization of aqueous wastes**

This case study was adapted from [12] and solved by [13] , who employed MEN's Grand Composition Curve (GCC) to find the cheapest MSA for the needed separation, Two phenol-rich streams (R_1 and R_2), two method MSAs (S_1 and S_2), and four potential external MSAs for phenol removal are present in the problem. These procedures include activated carbon adsorption, S3, polymeric resin ion exchange, S4, air stripping, S5, and light oil. [12] Indicated the first three external MSAs, while [14] stated the final. S_3 is the cheapest MSA for the needed separation. Table 8 shows stream data. Operating time each year is 8600 hours.

Table 8. Stream data for case study (2)

Rich stream						
stream		G_i		Y_i^{in}		Y_i^{out}
R_1		2		0.05		0.010
R_2		1		0.03		0.006
Lean stream						
Stream	L_j	X_j^{in}	X_j^{out}	m_j	b_j	C ($/kg$)
S_1	5	0.005	0.015	2	0	0
S_2	3	0.010	0.03	1.53	0	0
S_3	∞	0	0.11	0.02	0	0.081
S_4	∞	0	0.51	0.09	0	0.255
S_5	∞	0	0.029	0.04	0	0.06
S_6	∞	0.0013	0.015	0.001	0.001	0.01

Table 9 shows the equilibrium solubility data for phenol, gas oil, and lube oil may be correlated by the following relations:

Table 9. The Equilibrium relation and cost for case study (2)

Stream		**Equilibrium relation**
Gas oil	*(S_1)*	$Y_1 = 2\,X_1$
Lube oil	*(S_2)*	$Y_2 = 1.53\,X_2$

The excess capacity of the process MSA (S_1), the load to be removed by external MSA (S_2), and the number of plates in column are the results of using the current method for the various values of the minimum composition approach (ε), as indicated in table (10).

Table 10. Different values of $(S_1, S_2, N_{s)})$ for various minimum composition approaches

Minimum composition approach (ε)	S_1	S_2	Number of plates ($N_{s)}$)
0.00100	2.0793	0.9062	18
0.00080	2.1375	0.8212	22
0.00060	2.1945	0.7380	23
0.00040	2.2550	0.6496	29
0.00038	2.2058	0.6653	28
0.00035	2.2665	0.6330	30
0.00030	2.3050	0.5770	35
0.00010	2.3375	0.5292	47

For each function, we calculate the normalized decision matrix for different (ε). Table 11 summarizes the results.

Table 11. Normalized decision matrix of MEN at different (ε) for case study (2)

| Minimum composition approach (ε) | Normalized decision matrix | | | | |
	S_1 (μ_1)	S_2 (μ_2)	Number of plates ($N_{s)}$) (μ_3)	Min	max
0.00100	1.000000	0.000000	1.000000	0.00000	
0.00080	0.774593	0.225464	0.862069	0.225464	
0.00060	0.553834	0.446154	0.827586	0.446154	
0.00040	0.319520	0.680637	0.620690	0.319520	
0.00038	**0.510070**	**0.638992**	**0.655172**	**0.510070**	**0.510070**
0.00035	0.274981	0.724668	0.586207	0.274981	
0.00030	0.125871	0.873210	0.413793	0.125871	
0.00010	0.000000	1.000000	0.000000	0.000000	

The optimum solution is selected based on the greater value (**0.510070**), which corresponds to (ε) = **0.00038**. The mass transfer load at the exchange unit is shown by a value in enclosed spaces in Fig.3, illustrating the optimal network structure.

To illustrate the approach that is recommended for the case study, at optimal composition approach is chosen at (ε = **0.00038**), the composition relations between the rich and lean streams, as shown in Table 12.

Table 12. The compositional relationships of the rich and lean streams.

X_1	X_2	Y
0.02462	0.03230	0.05000
0.02286	0.03000	0.04648
0.03100	0.01972	0.03076
0.01462	0.01923	0.03000
0.00756	0.01000	0.01588
0.00500	0.00665	0.01076
0.00462	0.00616	0.01000
0.00262	0.00354	0.00600

The composition interval is used to calculate excess capacity and the load to be reduced. The mass transfer pinch is located by constructing a CID shown in Table 13.

Table 13. Composition interval diagram [CID] for case study (2) at ε = 0.00038.

From Table 13, Excess capacity of process = 0.01484 Kg/Sec, then the actual amount of $S_2 = (3 - \frac{0.01484}{(0.03-0.01)})$ =2.258 Kg/Sec, similar the load to be removed = 0.00884 Kg/Sec, then the amount of S_3 required = $\frac{0.00884}{(0.11-0)}$ =0.0804 Kg/Sec, at the pinch point the rich stream composition

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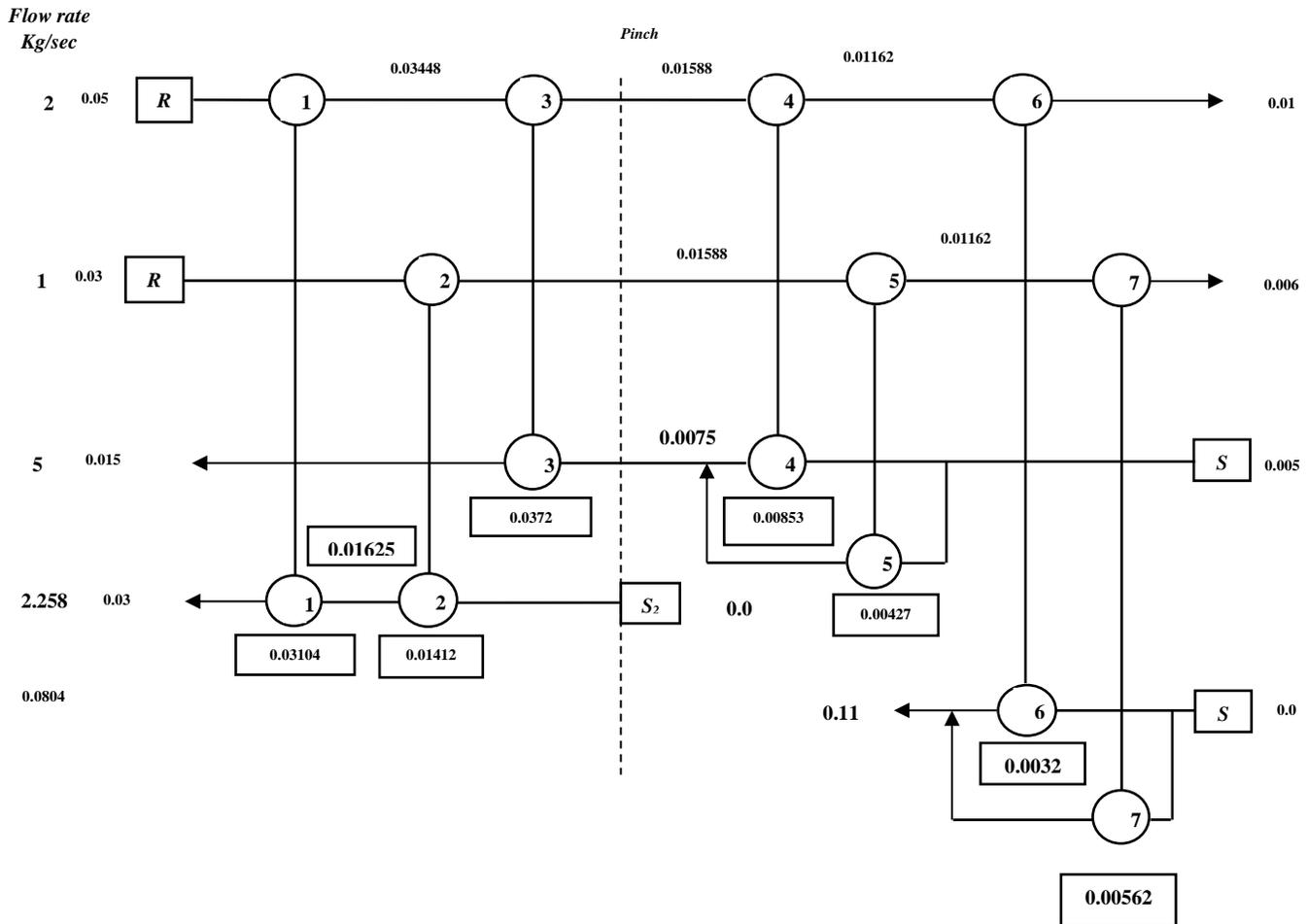
$= 0.01588$

Figure (3) the optimal network design of MEN for case 2

Comparison between different solutions is shown in Table 14, where [12] report a network cost of $(36.456* 10^4$ \$/Yr$)$ for minimum composition approach equal (0.001) using Pinch analysis method and [13] have obtained a network costing $((60.850* 10^4)$ \$/Yr$)$, for minimum composition approach equal (0.001) using MINNLP super structure, In this work, it was found that the cost optimal network has a minimum composition approach of (0.00038) and network cost equal to $((32.908 * 10^4)$ \$/Yr$)$ by using fuzzy method.

Table 14. Comparisons of solution obtained by different authors for case study (2)

Author	used techniques	Minimum composition approach (ε)	Number of stages (N_{actual})	Annual fixed cost ($/year)	Annual operating cost $*10^4$ ($/year)	Total annual cost $*10^4$ ($/year)
[12]	Pinch analysis	0.001	18	81936	28.262	36.456
[13]	MINNLP superstructure	0.001	28	127456	48.251	60.850
Current work	Fuzzy approach	0.00038	28	127456	20.162	32.908

IV. RESULTS AND DISCUSSION

The case studies for coke-oven gas (COG) sweetening and phenol removal demonstrate the benefits of the fuzzy logic-based multi-objective optimization method for synthesizing Mass Exchange Networks (MENs). Compared to traditional techniques like Pinch analysis and Mixed-Integer Non-Linear Programming (MINLP), the fuzzy approach provides more cost-effective, environmentally compliant, and flexible solutions.

In the COG sweetening study, the fuzzy method resulted in an optimal solution at a minimum composition approach (ε) of 0.00025, with a network cost of $42.257 * 10^4$/year. This was more cost-effective than the Pinch analysis ($52.604 * 10^4$/year) and MINLP ($91.80 * 10^4$/year) methods. The fuzzy approach not only minimized costs but also optimized environmental factors, such as pollutant removal and separation efficiency, which traditional methods couldn't handle within a single framework. Additionally, the fuzzy method demonstrated superior flexibility by managing multiple objectives at once, making it highly adaptable for real-world applications with large datasets and dynamic conditions.

In the phenol removal case study, the fuzzy method again outperformed both Pinch analysis and MINLP. At (ε = 0.00038), the fuzzy-based optimization resulted in a network cost of $32.908 * 10^4$/year, much lower than the Pinch ($36.456 * 10^4$/year) and MINLP ($60.850 * 10^4$/year) approaches. The ability of fuzzy logic to balance multiple objectives—such as cost, pollutant

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removal, and operational efficiency—enabled a more comprehensive solution for MEN design. A key advantage of the fuzzy approach is its flexibility in handling conflicting objectives simultaneously. Unlike Pinch analysis, which focuses solely on cost minimization, or MINLP, which can be computationally intense, fuzzy logic integrates environmental and operational factors seamlessly, making it well-suited for real-world scenarios.

Another benefit is scalability. As MENs grow more complex, the fuzzy approach performs more efficiently than MINLP, which struggles with the increasing computational demands of large systems. Additionally, fuzzy logic offers adaptability in dynamic environments, allowing for design adjustments in response to changing conditions without the need for full re-optimization. The fuzzy method provides a powerful, cost-effective, and scalable solution for MEN design. It balances both cost reduction and environmental compliance, offering a more holistic approach to optimization. The ability to handle multiple objectives, its computational efficiency, and its adaptability make it a promising alternative to more complex methods like Pinch analysis and MINLP. In summary, the fuzzy logic method's flexibility, efficiency, and scalability make it an excellent option for MEN synthesis, especially in dynamic industrial environments. Future research could explore integrating fuzzy logic with other optimization techniques, such as machine learning or genetic algorithms, to enhance its robustness and broaden its application in more complex industrial scenarios.

V. CONCLUSION

This research aims to explore multi-objective optimization for the design of mass exchange networks using a fuzzy logic methodology. The focus is primarily on minimizing the network's annual total cost by selecting the most suitable set of permissible composition differences. A multi-objective function was created to incorporate aspects such as the additional capacity of the process mass separation agent, the load to be removed by the external separation agent, and the number of plates in the column. The fuzzy logic approach was applied to determine the optimal network design, and the method was tested through two case studies. The results demonstrate that this approach yields more accurate solutions compared to those found in prior research. Additionally, the technique does not require any advanced mathematical or computational expertise, making it a practical and accessible tool for engineers. Overall, this research presents a novel and efficient process for optimizing mass exchange networks, offering engineers a method to design cost-effective and efficient systems.

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Nomenclature

MENs	Mass exchange networks
TAC	Total annualized cost
MINLP	Mixed integer nonlinear program
MILP	Mixed integer linear program
LP	Linear program
MSA	Mass separating agent
CID	Configuration interval diagram
R_I	Rich stream
G_i	Flow rate of the rich stream
S_J	Lean stream
L_J	Lean stream flow rate
AOC	Annual operating cost
M	Normalized decision matrix

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